ON THE ELASTIC EQUILIBRIUM OF A BODY LIMITED BY A CONE OF REVOLUTION

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The subject problem is analyzed for the case where the surface displacements are given using the theory of Green's functions. The case where the surface stresses are specified is briefly discussed.

1. *Case in which surface displacements are given.* Among the cases in /316** which the problem of elastic equilibrium is fairly simply solved by applying the usual principles which we have utilized many other times***, we have the problem of an isotropic body limited by a cone of rotation. In this report, we plan to give a rapid indication of the solution to this problem.

Let us remember that, if the displacements u, v, w are given on the surface of a deformed elastic isotropic body in equilibrium, the problem consists /317 of formulating the expression

$$u=\frac{1}{4\pi}\int_\sigma u\frac{dG}{dn}\,d\sigma-\frac{\lambda+\mu}{2\mu}x\theta+\frac{\lambda+\mu}{8\pi\mu}\int_\sigma \xi\theta\frac{dG}{dn}\,d\sigma, \tag{1}$$

and the analogous ones for v and w, on the assumption that harmonic function θ (elementary expansion) inherent in the problem is known for the time being. θ is determined in such a way that we have identically:

$$\begin{aligned}&\frac{3\lambda+5\mu}{2\mu}\theta+\frac{\lambda+\mu}{2\pi}\left(x\frac{\partial\theta}{\partial x}+y\frac{\partial\theta}{\partial y}+z\frac{\partial\theta}{\partial z}\right)-\\&-\frac{\lambda+\mu}{8\pi\mu}\left(\frac{\partial}{\partial x}\int_\sigma\xi\theta\frac{dG}{dn}\,d\sigma+\frac{\partial}{\partial y}\int_\sigma\eta\theta\frac{dG}{dn}\,d\sigma+\frac{\partial}{\partial z}\int_\sigma\zeta\theta\frac{dG}{dn}\,d\sigma\right)=\\&=\frac{1}{4\pi}\left(\frac{\partial}{\partial x}\int_\sigma u\frac{dG}{dn}\,d\sigma+\frac{\partial}{\partial y}\int_\sigma v\frac{dG}{dn}\,d\sigma+\frac{\partial}{\partial z}\int_\sigma w\frac{dG}{dn}\,d\sigma\right).\end{aligned} \tag{2}$$

The significance of the symbols in these formulas is the accustomed one: σ is the surface of the body; G, Green's function, the use of which, as in the references cited, is limited to expeditious representation of a harmonic function which acquires the given values on the surface; n, the internal normal to σ; λ and μ, Lamé's constants; and ξ, η, ζ are the values of x, y, z on σ.

Let us now assume that our cone has the x-axis as its axis of rotation and the coordinate origin as its apex. Let us restrict ourselves to consideration of the case in which the elastic body is limited to the segment of the

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* Presented by V. Volterra, Society member.

** Numbers in the margin indicate pagination in the original foreign text.

***Note: See, for example, *On the Problem of Elastic Equilibrium of an Indefinite Circular Cylinder*, Rendiconti della Reale Accademia dei Lincei, Vol. 13, 1st Half-year, p. 232, 1904, and *On the Problem of Elastic Rotation of an Ellipsoid of Rotation*, Ibid., Vol. 14, 1st Half-year, p. 76, 1905.

conic surface which, in its interior, contains the positive direction of the x-axis. However, we must immediately note, this solution is readily extended to the more general case in which the elastic body is limited in any fashion merely by conical surfaces of rotation which have the same apex and the same axis.

Let us introduce the system of curvilinear coordinates:

$$(3)\quad x = e^{l}t\ ,\ y = e^{l}\sqrt{1-t^2}\cos\psi\ ,\ z = e^{l}\sqrt{1-t^2}\,\text{sen}^{*}\psi\ ,\ 0 < t \leq 1. \tag{3}$$

The equation of the conical surface which delimits our body will then be $t = t_0$ with $0 < t_0 < 1$, and in order to pass through all the points of the elastic body it will suffice to vary l from $-\infty$ to $+\infty$, t from t_0 to $+1$, and ψ from 0 to 2π.

Let us now suppose that on the surface $t = t_0$, a function $\theta\ (l,\psi)$ is given which is subjected to the conditions that for $l = -\infty$, or at the apex of the cone -- whatever the generatrix is along which we move -- this function converges toward the same finite value, hence it is independent of ψ. For $l = +\infty$, it becomes zero, at least as e^{-2l}. On the rest of the conical surface, $\theta(l,\psi)$ is assumed to be developable into a Fourier series with respect to ψ, representable by the Fourier integral with respect to l and having finite derivatives with respect to the latter variable. Then we will be able to write the following: /318

$$\theta(l,\psi) = e^{-\frac{l}{2}}\sum_{0}^{\infty}{}_{m}\left[\theta_m(l)\cos m\psi + \bar{\theta}_m(l)\,\text{sen}\,m\psi\right], \tag{4}$$

and it will be seen from this that when $l = -\infty$ the values of θ_m, $\bar{\theta}_m$ will become zero at least according to $e^{\frac{l}{2}}$, and when $l = +\infty$ at least according to $e^{-\frac{3l}{2}}$. As is known, the harmonic and regular function in the interior of the cone, which takes on the values of expression (4) on its surface, will also be represented by the formula

$$\frac{e^{-\frac{l}{2}}}{2\pi}\sum_{0}^{\infty}{}_{m}\int_{-\infty}^{+\infty}\frac{K_m(t,i)}{K_m(t_0,i)}\,di\int_{-\infty}^{+\infty}\left[\theta_m(\varrho)\cos m\psi + \bar{\theta}_m(\varrho)\,\text{sen}\,m\psi\right]\cos i(l-\varrho)\,d\varrho, \tag{5}$$

where $K_m(t,i) = (1-t^2)^{\frac{m}{2}}\frac{d^m K_0(t,i)}{dt^m}$ and $K_0(t,i) = K(t,i)$ are the known Mehler function. This is defined, minus a constant factor, as the integral of the equation

$$(1-t^2)\frac{d^2K}{dt^2} - 2t\frac{dK}{dt} - \left(i^2+\frac{1}{4}\right)K = 0$$

which is finite when $t = +1$ and for any value of i. The constant factor is so chosen that

$$K(t,i) = F\left(\frac{1}{2}+i\sqrt{-1}\ ,\ \frac{1}{2}-i\sqrt{-1}\ ,\ 1\ ;\ \frac{1-t}{2}\right) =$$
$$= 1 + (4i^2+1)\frac{1-t}{2^3} + \frac{(4i^2+1)(4i^2+3^2)}{(2!)^2}\frac{(1-t)^2}{2^6} + \cdots$$
$$\cdots + \frac{(4i^2+1)(4i^2+3^2)\ldots(4i^2+(2r-1)^2)}{(r!)^2}\frac{(1-t)^r}{2^{3r}} + \cdots$$

* Translator's note: sen is properly sin in English terminology.

where the symbol F is the accustomed symbol of the hypergeometric series.

2. Now to solve our problem let us write:

$$\frac{1}{4\pi}\int_\sigma u\frac{dG}{dn}\,d\sigma = \frac{e^{-\frac{l}{2}}}{2\pi}\sum_0^\infty{}_m\int_{-\infty}^{+\infty}\frac{K_m(t,i)}{K_m(t_0,i)}\,di\times \tag{6}$$
$$\times\int_{-\infty}^{+\infty}[u_m(\varrho)\cos m\psi + \bar{u}_m(\varrho)\operatorname{sen} m\psi]\cos i(l-\varrho)\,d\varrho$$

and for $\frac{1}{4\pi}\int_\sigma v\frac{dG}{dn}\,d\sigma$, $\frac{1}{4\pi}\int_\sigma w\frac{dG}{dn}\,d\sigma$ let the similar expressions be valid which are obtained from the preceding by respectively setting the values $v_m(\rho)$, $\bar{v}_m(\rho)$; $w_m(\sigma)$; $\bar{w}_m(\rho)$ in place of $u_m(\rho)$, $\bar{u}_m(\rho)$. The values of these functions $u_m(\rho)$ ... /319 ... $\bar{w}_m(\rho)$ are assumed to be known and derived from the given values of u, v, w on σ, which we will assume to satisfy the general conditions enunciated in the preceding section. Let us now attempt to satisfy the problem by assuming the values which θ assumes on σ in the form (4) and hence the values themselves which θ takes on in the whole elastic body in the form (5). The feasibility of determining θ_m, $\bar{\theta}_m$ so that the assumptions advanced regarding θ are verified may be deducted from the *a posteriori* solution.

If, as we have said, the values of θ are given by expression (5), it is also immediately found that:

$$\left.\begin{aligned}
&\frac{1}{4\pi}\int_\sigma \xi\theta\frac{dG}{dn}\,d\sigma = \frac{t_0\,e^{-\frac{l}{2}}}{2\pi}\sum_0^\infty{}_m\int_{-\infty}^{+\infty}\frac{K_m(t,i)}{K_m(t_0,i)}\,di\times\\
&\qquad\times\int_{-\infty}^{+\infty}e^{\varrho}[\theta_m(\varrho)\cos m\psi + \bar{\theta}_m(\varrho)\operatorname{sen} m\psi]\cos i(l-\varrho)\,d\varrho,\\
&\frac{1}{4\pi}\int_\sigma \eta\theta\frac{dG}{dn}\,d\sigma = \frac{\sqrt{1-t_0^2}\,e^{-\frac{l}{2}}}{2\pi}\left\{\cos\psi\int_{-\infty}^{+\infty}\frac{K_1(t,i)}{K_1(t_0,i)}\,di\times\right.\\
&\qquad\times\int_{-\infty}^{+\infty}e^{\varrho}\theta_0(\varrho)\cos i(l-\varrho)\,d\varrho + \tfrac{1}{2}\sum_1^\infty{}_m\int_{-\infty}^{+\infty}\frac{K_{m+1}(t,i)}{K_{m+1}(t_0,i)}\,di\times\\
&\qquad\times\int_{-\infty}^{+\infty}e^{\varrho}[\theta_m(\varrho)\cos(m+1)\psi + \bar{\theta}_m(\varrho)\operatorname{sen}(m+1)\psi]\cos i(l-\varrho)\,d\varrho +\\
&\qquad+\tfrac{1}{2}\sum_1^\infty{}_m\int_{-\infty}^{+\infty}\frac{K_{m-1}(t,i)}{K_{m-1}(t_0,i)}\,di\int_{-\infty}^{+\infty}e^{\varrho}[\theta_m(\varrho)\cos(m-1)\psi +\\
&\qquad\left.+\bar{\theta}_m(\varrho)\operatorname{sen}(m-1)\psi]\cos i(l-\varrho)\,d\varrho\right\},\\
&\frac{1}{4\pi}\int_\sigma \zeta\theta\frac{dG}{dn}\,d\sigma = \frac{\sqrt{1-t_0^2}\,e^{-\frac{l}{2}}}{2\pi}\left\{\operatorname{sen}\psi\int_{-\infty}^{+\infty}\frac{K_1(t,i)}{K_1(t_0,i)}\,di\times\right.\\
&\qquad\times\int_{-\infty}^{+\infty}e^{\varrho}\theta_0(\varrho)\cos i(l-\varrho)\,d\varrho + \tfrac{1}{2}\sum_1^\infty{}_m\int_{-\infty}^{+\infty}\frac{K_{m+1}(t,i)}{K_{m+1}(t_0,i)}\,di\times\\
&\qquad\times\int_{-\infty}^{+\infty}e^{\varrho}[\theta_m(\varrho)\operatorname{sen}(m+1)\psi - \bar{\theta}_m(\varrho)\cos(m+1)\psi]\cos i(l-\varrho)\,d\varrho +\\
&\qquad+\tfrac{1}{2}\sum_1^\infty{}_m\int_{-\infty}^{+\infty}\frac{K_{m-1}(t,i)}{K_{m-1}(t_0,i)}\,di\int_{-\infty}^{+\infty}e^{\varrho}[-\theta_m(\varrho)\operatorname{sen}(m-1)\psi +
\end{aligned}\right\} \tag{7}$$

$$\left\{ \qquad + \bar{\theta}_m(\varrho)\cos(m-1)\psi]\cos i(l-\varrho)\,d\varrho \right\} \tag{7}$$

and all the terms which appear in expression (1) are formulated. It is now, as customarily, a question of determining θ_m, $\bar{\theta}_m$ so that expression (2) is likewise satisfied. Let us note that /320

$$\frac{\partial}{\partial x} = e^{-t}\left[t\frac{\partial}{\partial l} + (1-t^2)\frac{\partial}{\partial t}\right],$$
$$\frac{\partial}{\partial y} = e^{-t}\left[\sqrt{1-t^2}\left(\frac{\partial}{\partial l} - t\frac{\partial}{\partial t}\right)\cos\psi - \frac{\operatorname{sen}\psi}{\sqrt{1-t^2}}\frac{\partial}{\partial\psi}\right],$$
$$\frac{\partial}{\partial z} = e^{-t}\left[\sqrt{1-t^2}\left(\frac{\partial}{\partial l} - t\frac{\partial}{\partial t}\right)\operatorname{sen}\psi + \frac{\cos\psi}{\sqrt{1-t^2}}\frac{\partial}{\partial\psi}\right]$$

and hence:

$$x\frac{\partial\theta}{\partial x} + y\frac{\partial\theta}{\partial y} + z\frac{\partial\theta}{\partial z} = \frac{\partial\theta}{\partial l} = -\frac{\theta}{2} +$$

$$+\frac{e^{-\frac{l}{2}}}{2\pi}\sum_0^\infty{}_m\int_{-\infty}^{+\infty}\frac{K_m(t,i)}{K_m(t_0,i)}\,di\int_{-\infty}^{+\infty}[\theta'_m(\varrho)\cos m\psi + \bar{\theta}'_m(\varrho)\operatorname{sen} m\psi]\cos i(l-\varrho)\,d\varrho,$$

$$\frac{1}{4\pi}\left(\frac{\partial}{\partial x}\int_\sigma \xi\theta\frac{dG}{dn}\,d\sigma + \frac{\partial}{\partial y}\int_\sigma \eta\theta\frac{dG}{dn}\,d\sigma + \frac{\partial}{\partial z}\int_\sigma \zeta\theta\frac{dG}{dn}\,d\sigma\right)_{t=t_0} =$$

$$= e^{-\frac{l}{2}}\sum_0^\infty{}_m[\theta'_m(l)\cos m\psi + \bar{\theta}'_m(l)\operatorname{sen} m\psi] +$$

$$+\tfrac{3}{2}e^{-\frac{l}{2}}\sum_0^\infty{}_m[\theta_m(l)\cos m\psi + \bar{\theta}_m(l)\operatorname{sen} m\psi]\cdot$$

$$+\frac{t_0(1-t_0^2)}{2\pi}e^{-\frac{3l}{2}}\left\{\int_{-\infty}^{+\infty}\frac{\partial}{\partial t_0}\log\frac{K_0(t_0,i)}{K_1(t_0,i)}\,di\int_{-\infty}^{+\infty}e^{\varrho}\theta_0(\varrho)\cos i(l-\varrho)\,d\varrho +\right.$$

$$+\tfrac{1}{2}\sum_1^\infty{}_m\int_{-\infty}^{+\infty}\frac{\partial}{\partial t_0}\log\frac{K_m^2(t_0,i)}{K_{m+1}(t_0,i)\,K_{m-1}(t_0,i)}\,di\times$$

$$\left.\times\int_{-\infty}^{+\infty}e^{\varrho}[\theta_m(\varrho)\cos m\psi + \bar{\theta}_m(\varrho)\operatorname{sen} m\psi]\cos i(l-\varrho)\,d\varrho\right\}.$$

In the same fashion, the values taken on by the second term when $t = t_0$ are written in the form

$$e^{-\frac{3l}{2}}\sum_0^\infty{}_m[f_m(l)\cos m\psi + \bar{f}_m(l)\operatorname{sen} m\psi].$$

When we take into account the results obtained, when $t = t_0$, expression (2) gives us

$$\begin{cases} e^{l}\theta_0(l) - \dfrac{\lambda+\mu}{\lambda+3\mu}\dfrac{t_0(1-t_0^2)}{2\pi}\displaystyle\int_{-\infty}^{+\infty}\frac{\partial}{\partial t_0}\log\frac{K_0(t_0,i)}{K_1(t_0,i)}\,di\times \\ \qquad\qquad\times\displaystyle\int_{-\infty}^{+\infty}e^{\varrho}\theta_0(\varrho)\cos i(l-\varrho)\,d\varrho = \frac{2\mu}{\lambda+3\mu}f_0(l), \\ e^{l}\theta_m(l) - \dfrac{\lambda+\mu}{\lambda+3\mu}\dfrac{t_0(1-t_0^2)}{4\pi}\displaystyle\int_{-\infty}^{+\infty}\frac{\partial}{\partial t_0}\log\frac{K_m^2(t_0,i)}{K_{m+1}(t_0,i)\,K_{m-1}(t_0,i)}\,di\times \\ \qquad\qquad\times\displaystyle\int_{-\infty}^{+\infty}e^{\varrho}\theta_m(\varrho)\cos i(l-\varrho)\,d\varrho = \frac{2\mu}{\lambda+3\mu}f_m(l), \end{cases} \tag{8}$$

and the values of $\theta_m(l)$ satisfy similar equations. With the formula given in /321
the report cited on the circular cylinder, $e^{l}\theta_m(l)$, $e^{l}\bar{\theta}_m(l)$, and hence θ_m and $\bar{\theta}_m$, are derived from these equations, and the problem may be regarded as entirely solved. This procedure, as we have said in all similar cases, needs to be verified, but we shall postpone this to another occasion.

3. *Regarding the case in which the surface stresses are given.* If, as is customary, we indicate the components of the elemental rotation by $\bar{\omega}_1$, $\bar{\omega}_2$, $\bar{\omega}_3$ and if we write the following for the surface values

$$(\varpi_i)_{t=t_0} = e^{-\frac{i}{2}} \sum_0^\infty{}_m [\varpi_{i,m}(l)\cos m\psi + \bar{\varpi}_{i,m}(l)\operatorname{sen} m\psi], \quad i = 1, 2, 3, \tag{9}$$

while we assume that the values taken on by θ on the surface are always given by expression (4), we may immediately construct the harmonic and regular functions inside the cone: θ; $\bar{\omega}_1$, $\bar{\omega}_2$, $\bar{\omega}_3$. Then because

$$\cos nx = \sqrt{1-t_0^2}, \quad \cos ny = -t_0\cos\psi, \quad \cos nz = -t_0\operatorname{sen}\psi;$$

$$\frac{d}{dn} = \sqrt{1-t_0^2}\, e^{-i}\frac{\partial}{\partial t},$$

the values of u, v, w inside the cone will be given by the formulas

$$\left\{\begin{aligned}
u &= \frac{1}{8\pi\mu}\int_\sigma \mathrm{L}G_1\, d\sigma - \frac{\sqrt{1-t_0^2}}{8\pi}\int_\sigma \theta G_1\, d\sigma - \\
&\quad - \frac{t_0}{4\pi}\int_\sigma (\varpi_3\cos\psi - \varpi_2\operatorname{sen}\psi)\, G_1\, d\sigma - \frac{\lambda+\mu}{2\mu}x\theta - \\
&\quad - \frac{\lambda+\mu}{8\pi\mu}t_0\sqrt{1-t_0^2}\int_\sigma \frac{\partial\theta}{\partial t} G_1\, d\sigma, \\
v &= \frac{1}{8\pi\mu}\int_\sigma \mathrm{M}G_1\, d\sigma + \frac{t_0}{8\pi}\int_\sigma \theta\cos\psi\, G_1\, d\sigma - \\
&\quad - \frac{1}{4\pi}\int_\sigma (t_0\varpi_1\operatorname{sen}\psi + \varpi_3\sqrt{1-t_0^2})\, G_1\, d\sigma - \frac{\lambda+\mu}{2\mu}y\theta - \\
&\quad - \frac{\lambda+\mu}{8\pi\mu}(1-t_0^2)\int_\sigma \frac{\partial\theta}{\partial t}\cos\psi\, G_1\, d\sigma, \\
w &= \frac{1}{8\pi\mu}\int_\sigma \mathrm{N}G_1\, d\sigma + \frac{t_0}{8\pi}\int_\sigma \theta\operatorname{sen}\psi\, G_1\, d\sigma + \\
&\quad + \frac{1}{4\pi}\int_\sigma (\varpi_2\sqrt{1-t_0^2} + t_0\varpi_1\cos\psi)\, G_1\, d\sigma - \frac{\lambda+\mu}{2\mu}z\theta - \\
&\quad - \frac{\lambda+\mu}{8\pi\mu}(1-t_0^2)\int_\sigma \frac{\partial\theta}{\partial t}\operatorname{sen}\psi\, G_1\, d\sigma,
\end{aligned}\right. \tag{10}$$

where L, M, N are the given stresses and G_1 is Neumann's function. Expressions /322
(10) are nothing else than basic formulas (8) of the first Mémoire, *Essay on a General Theory, etc.**, in which our particular hypotheses were introduced. We may now always write:

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* Annali di Matematica, Vol. 8 of Series 3, p. 129.

$$\frac{1}{4\pi}\int_\sigma LG_1\,d\sigma = -\frac{e^{-\frac{l}{i}}}{2\pi\sqrt{1-l_0^2}}\sum_0^\infty{}^m\int_{-\infty}^{+\infty}\frac{K_m(l,i)}{\frac{\partial}{\partial l_0}K_m(l_0,i)}\,di\times$$

$$\times\int_{-\infty}^{+\infty}[L(\varrho)\cos m\psi+\bar{L}(\varrho)\operatorname{sen} m\psi]\cos i(l-\varrho)\,d\varrho \tag{11}$$

and two other similar formulas for $\frac{1}{4\pi}\int_\sigma MG_1\,d\sigma$, $\frac{1}{4\pi}\int_\sigma NG_1\,d\sigma$, which may be obtained from the preceding by respectively putting $M(\rho)$, $\bar{M}(\rho)$; $N(\rho)$, $\bar{N}(\rho)$ for $L(\rho)$, $\bar{L}(\rho)$. Starting from the values of expressions (4) and (9) for θ; $\bar{\omega}_1$, $\bar{\omega}_2$, $\bar{\omega}_3$ on the surface, we may easily formulate all the terms which appear in expression (10), and the complete solution to the problem is reduced to determining the unknown functions θ_m, $\bar{\theta}_m$; $\bar{\omega}_{i,m}$, $\bar{\bar{\omega}}_{i,m}$ from the equations

$$\left\{\begin{array}{c}\theta=\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}+\dfrac{\partial w}{\partial z};\\ 2\varpi_1=\dfrac{\partial w}{\partial y}-\dfrac{\partial v}{\partial z}\ ,\ 2\varpi_2=\dfrac{\partial u}{\partial z}-\dfrac{\partial w}{\partial x}\ ,\ 2\varpi_3=\dfrac{\partial v}{\partial x}-\dfrac{\partial u}{\partial y}.\end{array}\right. \tag{12}$$

A complete exposition of all the calculations would require more space than that granted us. Therefore, we will limit ourselves to noting the procedure. For more explicit clarification, refer to the report on the cylinder, which has been quoted several times. The method which is among the simplest for completing the solution consists of taking into consideration the equations:

$$(\lambda+2\mu)\frac{\partial\theta}{\partial x}+2\mu\left(\frac{\partial\varpi_2}{\partial z}-\frac{\partial\varpi_3}{\partial y}\right)=0 \tag{13}$$

and the two similar ones. By means of this equation (13) and of the two last in expression (12), the rotations are successfully eliminated from the first equation in expression (12), and then this equation determines θ_m, $\bar{\theta}_m$. By means of the two equations similar to equation (13), $\bar{\omega}_2$ and $\bar{\omega}_3$ may be eliminated from the second equation in expression (12), and this equation is then capable of determining $\bar{\omega}_{1,m}$, $\bar{\bar{\omega}}_{1,m}$. Finally, the other unknown functions $\bar{\omega}_{2,m}$ $\bar{\bar{\omega}}_{2,m}$; $\bar{\omega}_{3,m}$, $\bar{\bar{\omega}}_{3,m}$ are determined by the two equations similar to equation (13), which we have just mentioned.

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